

Chiral Cosmological Models: Dark Sector Fields Description

Chervon S.V.

*Laboratory of Gravitation, Cosmology, Astrophysics,
Ilya Ulyanov State Pedagogical University, Ulyanovsk 432700, Russia*

Abstract. The present review is devoted to a Chiral Cosmological Model as the self-gravitating nonlinear sigma model with the potential of (self)interactions employed in cosmology. The chiral cosmological model has successive applications in descriptions of the inflationary epoch of the Universe evolution; the present accelerated expansion of the Universe also can be described by the chiral fields multiplet as the dark energy in wide sense.

To be more illustrative we are often addressed to the two-component chiral cosmological model. Namely, the two-component chiral cosmological model describing the phantom field with interaction to a canonical scalar field is analyzed in details. New generalized model of quintom character is proposed and exact solutions are founded out.

In the review we represented the perturbation theory for chiral cosmological model with the aim to describe the structure formation using the progress achieved in the inflation theory. It was shown that cosmological perturbations from chiral fields can be decomposed for inflaton and the dark sector perturbations. The two-component model is investigated in details, the general solution for shortwave approximation is obtained and analyzed for power law Universe expansion.

New issue for understanding the features of Universe evolution is proposed by consideration of the dark sector fields on the inflaton background. The results are illustrated for the solutions in the long-wave approximation.

1 Introduction

Active investigations of the Early Universe have been originated with understanding of inflationary epoch necessity in the beginning of the 80-s last century. During the decade it became clear strong relation between particle physics (micro-physics) and cosmology (macro-physics) leading to connection of quantum fluctuations in the very early Universe with it large scale structure observed nowadays. Such topics as topological defects, inflation, dark matter, axions and quantum cosmology were studied by many scientists. The questions about physical roots of the Universe expansion source (inflation) or quantum creation of the Universe were discussed in the literature as well (for review see [1]).

The discovery of the current acceleration of the Universe at the end of the 20-th century leads to a great number of new theoretical hypothesis to explain this observational fact. We can think over Dark Energy (DE) as the source of modern accelerated expansion of the Universe in a wide sense. The most often DE is represented by the cosmological constant Λ , quintessence as a canonical scalar field with the potential or as the scalar fields coupled to other species of matter/radiation and many others (see, for example, [2]). The general term *dark sector fields* is used to cover all kinds of DE and Dark Matter (DM). let us study a chiral nonlinear sigma model coupled to gravity in order to make the general framework for dark sector fields and take into account some generalization for multiple-field model.

A self-gravitating Nonlinear Sigma Model (NSM) with a potential of (self)interactions has been applied for description of inflationary cosmology [3] and therefore was called *The Chiral Inflationary Model*. Two years earlier pure kinetic NSM has been considered in cosmology [4], but it could not carry all necessary properties of inflationary period because the equation of state was corresponded to ultra-stiff matter only.

The term "chiral" is considered in the sense of short equivalent to the general chiral NSM with a Riemannian metric as a target space in accordance with terminology in the review [5]. Paradoxically, this term has been applied to pure bosonic NSM which has no relation to chiral symmetry because pure bosonic NSM does not contain the spinor fields. Thus the term "chiral" means that the scalar fields are not free and take values on some nonlinear manifold.

1.1 NSM in Cosmology

The chiral cosmological model (CCM) is based on the NSM with the potential of (self)interactions and CCM are generalization for a theory of scalar field singlet which has a lot of applications in cosmology. Therefore it is of interest to consider cosmological applications of the NSM.

NSM coupled to the gravitational field has been introduced from the different positions. The 4-dimensional NSM as a generalization from 2-dimensional one can be presented only with coupling to the gravitational field if one wants to have desirable analogy with the gauge theories expressed in existence of instanton and meron solutions. This approach (based on a Riemannian space-time with Euclidian signature) has been realized by De Alfaro et al [6].

Considering the NSM as the source of the gravitational field (based on a Riemannian space-time with Lorentzian signature) George Ivanov [7] independently of work [6] proposed the self-gravitating NSM as a generalization from multiple scalar fields theory. In the work [7] the basic equations of the self-gravitating NSM were derived and the methods of solving them were suggested. The applications of the suggested methods for spherical-symmetric, plane-symmetric space-times and for cosmological metrics have been demonstrated and examples of exact solutions were obtained. The cosmological solutions include: A) $SO(4)$ -invariant solution for closed Friedmann – Robertson – Walker Universe with a constant scale factor $a(t) = a_0 = \text{const}$ (a static universe) or with $a(t) \propto t$ with effective Equation of State (EoS) corresponding to the radiation $p = -\frac{2}{3}$; B) $SO(3)$ -invariant solution for an anisotropic universe.

Let us note here that mentioned above cosmological solutions have been obtained with the isometric embedding method also proposed for the self-gravitating NSM in the work [7].

Further applications of NSM in cosmology were started from the works [8],[4]. In parallel with our investigations (Chervon et al, 1992-2012) there were few works on the NSM applications in cosmology. In the work [9] the evolution of perturbations to the flat FRW universe has been considered for $O(N)$ nonlinear sigma model at large N . The feature of the proposed model was the inclusion into consideration the "stiff source" which does not affect on the background determined by multicomponent perfect fluid. The effect of the "stiff source" is compared with metric and density of fluid perturbations; the stiff source may lead to initially non-Gaussian distribution on scales of several hundreds megaparsecs. Further extension of this model for the inflationary epoch and analysis of influence of the stiff source on the large scale structure formation was presented at the work [10].

Multiple scalar fields coupled to gravity have been considered in the framework of scalar-tensor theories of gravitation in the work [11]. It was stated out that for realization of the exact solar system tests it was necessary that some scalar kinetic terms should be non-positive-definite. This statement means the existence of phantom fields in the Universe.

The spherically-symmetric, static solutions to the $SU(2)$ NSM on de Sitter background have been studied by numerical methods in the work [12]. It was found that the countable set of regular solutions with finite energy exists there.

1.2 Coupled dark sector fields

After the discovery of the Universe acceleration the concept of Dark Energy (DE) has been actively studied. First of all DE is considered as the cosmological constant and the scalar field (quintessence, phantom, tachyon etc.). Other presentations of DE include modified gravity, Chaplygin gases and many other models [2].

The goal of our work is to generalize the models with interacting species of the Universe of the scalar field type and to describe them in terms of the chiral cosmological model. We will not pay much attention here to interaction of the scalar field with cosmic fluid or dust/radiation/ordinary matter.

The authors of the work [13] proposed the new cosmological model as a unified picture of quintessence and new form of DM. This, so called Frustrated CDM model, deals with two gravitationally interacting scalar fields without kinetic and potential interaction. Further application of this model to reconstruction quintessence from supernova observations is considered in [14].

It was proposed in the works [15], [16] the cosmological model in which DM and DE are modelled by two gravitationally coupled scalar fields. This model contains 95% of scalar fields. The shape of the potentials was chosen in specific form which gave opportunity to define the scalar DM mass (of order 10^{-26} eV). The implication on the structure formation was analyzed and the minimal cut-off radius ($r_c \sim 1.2$ kpc) for this model was obtained.

The models with a nonminimal coupling between DM and scalar field DE were investigated in the article [17]. The evolution of linear perturbations for such models was considered in details. It should be mention that proposed nonminimal coupling may lead for kinetic interaction if DM specie will be presented as the scalar field.

It was first time mention [18] about the fact that the same scalar field can play the role of early-time inflaton and later-time DE. The (dark) matter can be included into consideration by an ordinary way as a cosmic fluid.

The article [19] is devoted to observation effects from quintom models. It should be noted that in special cases quintom models can be presented with two gravitationally and potentially interacting scalar fields.

Investigation of the dynamic interaction between DE and DM have been analyzed in [20]. The general coupling between DE and DM is described by the additional term in the energy balance equation. This type of interaction can be extended for two gravitationally coupled scalar fields if DM will be presented as a scalar field. Note that energy transfer DM to DE and DE to DM was considered in the work [21] as well.

The model with N -dimensional internal space for a multi-field configuration was proposed in the work [22]. The difference from the chiral cosmological model [3], [23] is in the restriction of the internal space metric for the constant one.

The work [24] is devoted to quintom model with $O(N)$ symmetry. The model is of importance because it presents the nonlinear sigma model with deformed chiral space [3], [23]. Moreover the article [24] may be

considered as the next step from the work [22] on the way to the chiral cosmological models (CCM): it presents CCM with the special choice of the internal space metric.

The dynamical properties of DE model with two gravitationally, kinetically and potentially coupled scalar fields were studied in the work [25]. The authors started from consideration of two-fields model with non-canonical kinetic term. This model is more wide then CCM and can be reduced for it under simple choice of the action. In the mentioned article the focus was exactly on the described situation: the chosen model was equivalent to the 2-component CCM with the cross interaction between scalar (chiral) fields. Further extension of the work [25] is presented in [26].

2 Chiral cosmological model

The action of CCM as the the action of the self-gravitating nonlinear sigma model (NSM) with the potential of (self)interactions $V(\varphi)$ [3], [23] and cosmological constant Λ reads:

$$S = \int \sqrt{-g} d^4x \left(\frac{R - 2\Lambda}{2\kappa} + \frac{1}{2} h_{AB}(\varphi) \varphi_{,\mu}^A \varphi_{,\nu}^B g^{\mu\nu} - V(\varphi) \right), \quad (1)$$

where R being a scalar curvature of a Riemannian manifold with the metric $g_{\mu\nu}(x)$, κ – Einstein gravitational constant, $\varphi = (\varphi^1, \dots, \varphi^N)$ being a multiplett of the chiral fields (we use a notation $\varphi_{,\mu}^A = \partial_\mu \varphi^A = \frac{\partial \varphi^A}{\partial x^\mu}$), h_{AB} being the metric of the target space (the chiral space) with the line element

$$ds_\sigma^2 = h_{AB}(\varphi) d\varphi^A d\varphi^B, \quad A, B, \dots = \overline{1, N}. \quad (2)$$

The energy-momentum tensor for the model (1) reads

$$T_{\mu\nu} = \varphi_{A,\mu} \varphi_{,\nu}^A - g_{\mu\nu} \left(\frac{1}{2} \varphi_{,\alpha}^A \varphi_{,\beta}^B g^{\alpha\beta} h_{AB} - V(\varphi) \right). \quad (3)$$

The Einstein equation can be transformed to

$$R_{\mu\nu} = \kappa \{ h_{AB} \varphi_{,\mu}^A \varphi_{,\nu}^B - g_{\mu\nu} V(\varphi) \} - \Lambda g_{\mu\nu} \quad (4)$$

which simplify the derivation of gravitational dynamics equations.

The standard form of the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G \quad (5)$$

will be also under consideration.

Varying the action (1) with respect to φ^C , one can derive the dynamic equations of the chiral fields

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \varphi_{,\mu}^A) - \frac{1}{2} \frac{\partial h_{BC}}{\partial \varphi^A} \varphi_{,\mu}^C \varphi_{,\nu}^B g^{\mu\nu} + V_{,A} = 0, \quad (6)$$

where $V_{,A} = \frac{\partial V}{\partial \varphi^A}$. Considering the action (1) in the framework of cosmological spaces, we arrive to a chiral (inflationary or) cosmological model [3], [23], [28], [29].

2.1 The two-component chiral cosmological model

To connect our consideration with dark sector fields interaction models presented in the Introduction let us start from analysis of the two-component CCM. The target space metric we select in the most general form taking into account nondiagonal metric component h_{12}

$$ds_\sigma^2 = h_{11}(\phi, \psi) d\phi^2 + 2h_{12}(\phi, \psi) d\phi d\psi + h_{22}(\phi, \psi) d\psi^2. \quad (7)$$

Thus we have two component chiral cosmological model as the source of the gravitational field with the target space metric (7) and two chiral fields denoted as

$$\varphi^1 = \phi, \quad \varphi^2 = \psi.$$

In terms of chosen target space (7) the energy-momentum tensor (3) can be presented in the following form

$$T_{\mu\nu} = h_{11} \phi_{,\mu} \phi_{,\nu} + 2h_{12} \phi_{,\mu} \psi_{,\nu} + h_{22} \psi_{,\mu} \psi_{,\nu} - g_{\mu\nu} \left[\frac{1}{2} h_{11} \phi_{,\rho} \phi^{,\rho} + h_{12} \phi_{,\rho} \psi^{,\rho} + \frac{1}{2} h_{22} \psi_{,\rho} \psi^{,\rho} - V(\phi, \psi) \right]. \quad (8)$$

The metric of homogeneous and isotropic Universe we take in the Friedman – Robertson – Walker (FRW) form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \epsilon r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right). \quad (9)$$

The Einstein equation with $\Lambda = 0$ can be represented in the form

$$H^2 = \frac{\kappa}{3} \left[\frac{1}{2} h_{11} \dot{\phi}^2 + h_{12} \dot{\phi} \dot{\psi} + \frac{1}{2} h_{22} \dot{\psi}^2 + V(\phi, \psi) \right] - \frac{\epsilon}{a^2}, \quad (10)$$

$$\dot{H} = -\kappa \left[\frac{1}{2} h_{11} \dot{\phi}^2 + h_{12} \dot{\phi} \dot{\psi} + \frac{1}{2} h_{22} \dot{\psi}^2 \right] + \frac{\epsilon}{a^2}, \quad (11)$$

where the overdot means the derivative with respect to cosmic time t .

The chiral fields equations for the model (7) in FRW universe (86) reads

$$3H \left(h_{11} \dot{\phi} + h_{12} \dot{\psi} \right) + \partial_t \left(h_{11} \dot{\phi} + h_{12} \dot{\psi} \right) - \frac{1}{2} \frac{\partial h_{11}}{\partial \phi} \dot{\phi}^2 - \frac{\partial h_{12}}{\partial \phi} \dot{\phi} \dot{\psi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \phi} \dot{\psi}^2 + \frac{\partial V}{\partial \phi} = 0, \quad (12)$$

$$3H \left(h_{12} \dot{\phi} + h_{22} \dot{\psi} \right) + \partial_t \left(h_{12} \dot{\phi} + h_{22} \dot{\psi} \right) - \frac{1}{2} \frac{\partial h_{11}}{\partial \psi} \dot{\phi}^2 - \frac{\partial h_{12}}{\partial \psi} \dot{\phi} \dot{\psi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \psi} \dot{\psi}^2 + \frac{\partial V}{\partial \psi} = 0. \quad (13)$$

Let us mention that one of the Einstein equations – Friedmann equation (10) can be obtained by integrating the linear combination of the fields equations $\dot{\phi}$ (12) + $\dot{\psi}$ (13) using the second Einstein equation (11).

If the law of Universe evolution $a = a(t)$ is specified the system of equations (10)-(13) can be considered as the system of differential equations of the second order with three unknown functions: two chiral fields ϕ and ψ , and the potential V . According with the method of fine tuning of the potential [39] we consider the scalar factor as given function on cosmic time : $a = a(t)$. The metric of a target space we will not fix as it traditionally accepted in HEP, leaving a freedom of adaptation to resolving problem. Making simple algebraic conversion of Einstein equations (10)-(11) one can find their useful implication:

$$\frac{1}{2} h_{11} \dot{\phi}^2(t) + h_{12} \dot{\phi}(t) \dot{\psi}(t) + \frac{1}{2} h_{22} \dot{\psi}^2(t) = \frac{1}{\kappa} \left[\frac{\epsilon}{a^2} - \dot{H} \right], \quad (14)$$

$$V(t) = \frac{3}{\kappa} \left(H^2 + \frac{1}{3} \dot{H} + \frac{2}{3} \frac{\epsilon}{a^2} \right). \quad (15)$$

For the sake of completeness let us present the consequence of the Einstein equation in the form often used for analysis of Universe acceleration parameter

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{3} \left[\frac{1}{2} h_{11} \dot{\phi}^2(t) + h_{12} \dot{\phi}(t) \dot{\psi}(t) + \frac{1}{2} h_{22} \dot{\psi}^2(t) - V \right]. \quad (16)$$

Let us turn our attention to the models with two scalar fields and possible kinetic interactions between them. This models are often called as *multicomponent or multiple-field inflationary* models which clearly [30] are contained into *chiral cosmological models*. Multiple-fields models describing dark sector fields we can divide into the following classes.

I Class – the models with two canonical scalar fields with different physical meaning of the fields. *II Class* – quintom models with one phantom and one canonical scalar fields. *III Class* – are the models with kinetic interactions between scalar fields. It is clear that the *I Class* can be described with $h_{11} = 1$, $h_{22} = 1$, $h_{12} = 0$; the *II Class* can be described with $h_{11} = 1$, $h_{22} = -1$, $h_{12} = 0$ and the *III Class* can be described with $h_{11} = 1$, $h_{22} = 1$, $h_{12} = f(\phi, \psi)$. Let us note that the case with $h_{11} = 1$, $h_{22} = -1$, $h_{12} = f(\phi, \psi)$ has not been considered in the literature yet.

On the other hand we have possibility to analyze the target space metric h_{AB} as the two dimensional surface Σ embedding into appropriate three dimensional space. Then in according with the surface's points classification we arrive to the following three classes. Namely, if the determinant of the chiral space metric $h = \det(h_{AB}) = h_{11}h_{22} - h_{12}^2$ has positive sign: $h > 0$ at the point $M \in \Sigma$, we have the *elliptic* point; if $h < 0$ – the *hyperbolic* point and if $h = 0$ with at least one from metric coefficient $h_{AB} \neq 0$ we have the *parabolic* point.

Thus our simple division above may be characterized by elliptic points (I-st class), hyperbolic points (II-nd class). The III-d class requests an additional investigation. If we consider the case with $h = 0$ and *all metric components does not equal to zero* we can reduce the model to one canonical field as it was done in the work [25] for the case $h_{11} = 1$, $h_{22} = 1$, $h_{12} = \frac{a}{2}$ (with $a = \text{constant}$) when $a = \pm 2$. Namely, we can introduce new field Θ by the relation

$$\Theta = \sqrt{h_{11}} \phi \pm \sqrt{h_{22}} \psi + \Theta_*, \quad (\Theta_* = \text{const}).$$

Then the cosmological dynamics will be reduced to the self-gravitating scalar singlet Θ .

Summing up the arguments above we can conclude that the chiral cosmological model gave for us good possibility not only describe the classes investigated in the literature mentioned above but also to consider the evolution of kinetic interactions between scalar fields. To this end we may introduce the dependance of the chiral metric components on the scalar fields: $h_{11} = h_{11}(\phi, \psi)$, $h_{12} = h_{12}(\phi, \psi)$, $h_{22} = h_{22}(\phi, \psi)$. Thus we make generalization of the multi-fields models and now, using the freedom in the choice of the chiral metric, we can search for evolutionary solutions and compare them with observation data. Let us also note that the investigation of possible functional dependence of the chiral metric components may lead to better understanding of physically important scalar fields governing the Universe evolution.

2.2 The generalized quintom models

Let us consider new parametrization of the quintom model. Namely, let us set

$$h_{11} = -1, \quad h_{22} = h_{22}(\phi), \quad h_{12} = 0,$$

where h_{22} is a given function on the first chiral field ϕ . This approach means that we have fixed the metric of internal (chiral) space, or by other words – the symmetry of the chiral space. The original chiral nonlinear sigma model belongs to this category as having $SU_2 \times SU_2$ symmetry.

Let us consider quintom analog to $SO(3)$ nonlinear sigma model, which has (in two dimensions) an interesting properties making the analogy with gauge theories: instanton and meron solutions, asymptotical freedom etc. To derive the system of equations (10)-(13) for quintom generalization of $SO(3)$ NSM we choose the following chiral metric components:

$$h_{11} = -1, \quad h_{22} = \sin^2 \phi, \quad h_{12} = 0. \quad (17)$$

After the substitution one can obtain the following system of equations

$$H^2 = \frac{\kappa}{3} \left[-\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \sin^2 \phi \dot{\psi}^2 + V(\phi, \psi) \right] - \frac{\epsilon}{a^2}, \quad (18)$$

$$\dot{H} = -\kappa \left[-\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \sin^2 \phi \dot{\psi}^2 \right] + \frac{\epsilon}{a^2}, \quad (19)$$

$$-\ddot{\phi} - 3H\dot{\phi} - \frac{1}{2} \sin 2\phi \dot{\psi}^2 + \frac{\partial V}{\partial \phi} = 0, \quad (20)$$

$$\ddot{\psi} + 3H\dot{\psi} + 2 \cot \phi \dot{\phi} \dot{\psi} + \sin^{-2} \phi \frac{\partial V}{\partial \psi} = 0. \quad (21)$$

To simplify the system above let us consider a spatially-flat Universe with $\epsilon = 0$. Also we may suggest that the potential of (self)interaction can be considered as the constant during some period of Universe evolution in analogy with inflationary period, when the potential is considered as plane. From the other hand the same situation we can consider as kinetic model but with cosmological term. Thus we denote $V = \text{const} = \Lambda$. Also let us put the gravitational constant κ equal to unity: $\kappa = 1$. After that one can reduce from combination of the equations (18) and (19) the equation on Hubble parameter H :

$$3H^2 + \dot{H} = \Lambda. \quad (22)$$

The well-known solutions are

$$H = \sqrt{\frac{\Lambda}{3}} \tanh(\sqrt{3\Lambda}t), \quad (23)$$

$$a = a_* [\cosh(\sqrt{3\Lambda}t)]^{1/3}. \quad (24)$$

Besides, if we request the density $\rho = K + V = K + \Lambda$ should be positive then $\Lambda > 0$ and $\dot{H} < \Lambda$. The last expression is always hold for the solution (24).

Let us consider the evolution of equation of state

$$\omega_{DS} = \frac{K - \Lambda}{K + \Lambda}.$$

Using the equations (14)-(15) and the solution (24) we can obtain the following asymptotes

$$\omega_{t \rightarrow 0} \rightarrow -\infty, \quad \omega_{t \rightarrow \infty} \rightarrow -1.$$

Thus we can state the strong phantom influence at the early stage of the Universe evolution and domination of DE at the late stages.

Integrating the equation (21) one can obtain the relation

$$\dot{\psi} = \frac{C_1}{\cosh(\sqrt{3\Lambda}t) \sin^2 \phi}, \quad C_1 = \text{const}. \quad (25)$$

With this relation we can integrate equation (20). The solution for the first (phantom) field can be obtained from the expression

$$\cos \phi = -\frac{\sqrt{C_1^2 + 2\Lambda}}{\sqrt{2\Lambda}} \sin \left(\sqrt{\frac{2}{3}} \arctan \left(\sinh(\sqrt{3\Lambda}t) \right) + C_2 \right). \quad (26)$$

It is worth to note that the solution (26) gives some restriction for the constant C_1 .

The second (canonical) field can be obtained from (25). The solution is

$$\psi - \psi_0 = \frac{1}{2a_*} \left[\ln \left| \frac{C_1}{\sqrt{2\Lambda}} \tan z + 1 \right| - \ln \left| \frac{C_1}{\sqrt{2\Lambda}} \tan z - 1 \right| \right], \quad (27)$$

$$z = \sqrt{\frac{2}{3}} \arctan(\sinh(\sqrt{3\Lambda}t)) + C_2. \quad (28)$$

With the same approach we can find the solutions for $h_{22} = \sinh^2 \phi$ and other interesting cases including the changing of the phantom and canonical fields in the chiral metric [31]. For example the case with $h_{22} = \phi^2$ gives the analytic solution

$$\phi = \pm \sqrt{2z^2 - \frac{C_1^2}{2a_*^3\Lambda}}, \quad z = \arctan(\sinh(\sqrt{3\Lambda}t)) + C_2, \quad (29)$$

$$\psi = \frac{a_*^3}{2} \left[\ln \left(\frac{C_1^2 \sqrt{3}}{2a_*^3 \sqrt{\Lambda}} - z \right) - \ln \left(\frac{C_1^2 \sqrt{3}}{2a_*^3 \sqrt{\Lambda}} + z \right) \right]. \quad (30)$$

Thus we can see that the gravitational field (24) admits a set of solutions with fixed h_{22} as the given function of ϕ . More wide class of solutions will be presented in the separate article [31].

3 Cosmological perturbations as quantum fluctuations for CCM

Inflationary paradigm is represented by the assertion that before the epoch of primordial nucleosynthesis the Universe expansion was accelerated¹. The existence of such inflationary period gives possibility to solve the flatness, the horizon and the monopole problems of the standard Big Bang cosmology [1]. Together with this advantage inflation can explain the production of the first density perturbations in the early Universe which are seeds to formation of large scale structure and for CMB temperature anisotropies. The primordial density perturbations are created from quantum fluctuations of the scalar field (inflaton), which is the source of the early inflation. Let us mention here that the existence of inflationary period is strongly confirmed by observations from COBE satellite and WMAP mission (for recent review see [32]).

Our purpose now is to extend the perturbation theory from scalar singlet (inflaton) to scalar multiplett represented by CCM. To this end let us follow by the basic idea of inflation in respect to large scale structure formation. Namely we suggest that quantum fluctuations of the chiral fields as well as metric fluctuation of the internal space in CCM lead to cosmological inhomogeneities in the same manner as the inflaton singlet fluctuation leads to primordial density perturbations, which finally lead to Universe structure. On this way we can use the standard results for gravitational perturbations which can be divided for scalar, vector and tensor types. Our task then will be to calculate the perturbations from energy momentum tensor of CCM.

Background equations for the chiral cosmological model are presented at the section 2.

It is well known that vector and tensor perturbations could not lead to instability. While the scalar perturbations give the growing inhomogeneities which effect to the dynamics of matter. Therefore much attention will be payed to the scalar perturbations which can lead to the structure formation.

3.1 General equations for chiral fields perturbations

Let us consider the perturbations of the gravitational field with the metric $g_{\alpha\beta}$ in a general form

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \delta g_{\alpha\beta}(\mathbf{x}, t), \quad \tilde{g}^{\alpha\beta} = g^{\alpha\beta} - \delta g^{\alpha\beta}(\mathbf{x}, t). \quad (31)$$

Here $g_{\alpha\beta}(x)$ is the background metric, $\tilde{g}_{\alpha\beta}$ is a perturbed metric, $\delta g_{\alpha\beta}$ is a metric perturbation. Note that $\delta g_{\alpha\beta}$ can be presented as scalar, vector or tensor perturbations of the gravitational field [33].

To find the equations for the perturbations of the first order we can use the following approximation

$$\tilde{\varphi}^A = \varphi^A + \delta \varphi^A(\mathbf{x}, t), \quad (32)$$

$$\tilde{h}_{AB}(\tilde{\varphi}^C) \equiv h_{AB}(\tilde{\varphi}^C) = h_{AB}(\varphi^C) + h_{AB,C} \delta \varphi^C. \quad (33)$$

The last expression (33) means that the chiral metric perturbations are due to chiral fields perturbations only, not from perturbations internal metric form. From (33) one can calculate the derivative with respect to spacetime coordinates with the formula

$$\partial_\alpha h_{AB}(\tilde{\varphi}^C) = h_{AB,C} \varphi_{,\alpha}^C + h_{AB,CD} \varphi_{,\alpha}^D \delta \varphi^C \quad (34)$$

To derive perturbed equations for chiral fields dynamic let us transform the fields equations (6) to the following view

¹ In respect to present Universe acceleration there are division for early inflation and later inflation

$$\frac{1}{2}g_{,\alpha}g^{\alpha\beta}h_{AB}\varphi_{,\beta}^B + g\partial_\alpha\left(g^{\alpha\beta}h_{AB}\varphi_{,\beta}^B\right) - \frac{1}{2}gh_{BC,A}\varphi_{,\alpha}^B\varphi_{,\beta}^Cg^{\alpha\beta} + g\partial_AV(\varphi) = 0. \quad (35)$$

Here $g = \det(g_{\alpha\beta})$.

The Einstein equation (5) for small perturbations linearized about background metric $g_{\mu\nu}$ takes the form

$$\delta G_\nu^\mu = \kappa\delta T_\nu^\mu. \quad (36)$$

Background components of the energy-momentum tensor are presented in the formula (3). Variation $\delta T_\nu^\mu = \tilde{T}_\nu^\mu - T_\nu^\mu$ can be reduced to the form

$$\begin{aligned} \delta T_\nu^\mu = & \delta\varphi_{,\nu}^A\varphi_A^\mu + \delta\varphi_{,\alpha}^B\{\varphi_{B,\nu}g^{\alpha\mu} - \delta_\nu^\mu\varphi_B^{\alpha\mu}\} + \delta\varphi^C\{h_{AB,C}\varphi_{,\nu}^A\varphi_{,\alpha}^Bg^{\mu\alpha} - \delta_\nu^\mu\frac{1}{2}h_{AB,C}\varphi_{,\alpha}^A\varphi_{,\beta}^Bg^{\beta\alpha} - \delta_\nu^\mu V_{,C}\} \\ & - \delta g^{\mu\alpha}h_{AB}\varphi_{,\nu}^A\varphi_{,\alpha}^B + \frac{1}{2}\delta g^{\alpha\beta}\delta_\nu^\mu h_{AB}\varphi_{,\alpha}^A\varphi_{,\beta}^B. \end{aligned}$$

Using relations (31)-(33) one can decompose the chiral fields equations (6) into background equations and perturbed ones. In accordance with our notations (31) and (32) the background equations take the same form as the equations (6). The equations for the chiral fields perturbations can be transformed to the form

$$\begin{aligned} & \delta\varphi_{,\beta\alpha}^Bg^{\alpha\beta}h_{AB} + \delta\varphi_{,\beta}^B\{\frac{1}{2}g_{,\alpha}g^{\alpha\beta}h_{AB} + gg_{,\alpha}^{\alpha\beta}h_{AB} + gh_{AC,B}g^{\alpha\beta}\varphi_{,\alpha}^C + gg^{\alpha\beta}h_{AB,C}\varphi_{,\alpha}^C - gh_{BC,A}\varphi_{,\alpha}^Cg^{\alpha\beta}\} + \\ & \delta\varphi^C\left\{\left(\frac{1}{2}g_{,\beta}^{\alpha\beta} + gg_{,\alpha}^{\alpha\beta}\right)h_{AB,C}\varphi_{,\beta}^B + gg^{\alpha\beta}h_{AB,DC}\varphi_{,\alpha}^D\varphi_{,\beta}^B + gg^{\alpha\beta}h_{AB,C}\varphi_{,\beta\alpha}^B - \frac{1}{2}gg^{\alpha\beta}h_{DB,AC}\varphi_{,\alpha}^B\varphi_{,\beta}^D - gV_{,AC}\right\} - \\ & \delta g^{\alpha\beta}\{gh_{AB,C}\varphi_{,\alpha}^C\varphi_{,\beta}^B + gh_{AB}\varphi_{,\alpha\beta}^B - \frac{1}{2}gh_{BC,A}\varphi_{,\alpha}^B\varphi_{,\beta}^C + \frac{1}{2}g_{,\alpha}h_{AB}\varphi_{,\beta}^B\} - \delta g_{,\alpha}^{\alpha\beta}gh_{AB}\varphi_{,\beta}^B + \frac{1}{2}\xi_{,\alpha}gg^{\alpha\beta}h_{AB}\varphi_{,\beta}^B = 0. \end{aligned}$$

Here $\xi = g^{\alpha\beta}\delta g_{\alpha\beta}$.

Presented in this section formulas solve the problem of construction the theory for scalar, vector and tensor type perturbations in the chiral cosmological model. It should be mentioned here that the solutions for the perturbations of the gravitational field (left hand side of the equation (36)) are independent from the source and have been obtained by Lifshitz [33].

Let us turn our attention to the scalar perturbations of the gravitational field and the perturbations of the chiral fields.

4 Perturbations for CCM in FRW Universe

To present the equations for cosmological perturbations we will use the conformal time η instead of cosmic time t as it is generally excepted (see, for example, review by Mukhanov et al [27]). First of all let us rewrite the background Einstein and chiral field equations in the conformal time defined by the relation $d\eta = \frac{dt}{a(t)}$.

4.1 Background equations

Let us consider now the chiral cosmological model (1) in the framework of homogeneous and isotropic FRW Universe with the metric (86). The background Einstein equations (5) in terms of the conformal time take the following form

$$3\{\mathcal{H}^2 + \epsilon\} = \kappa\left\{\frac{1}{2}h_{AB}\varphi_{,\eta}^A\varphi_{,\eta}^B + a^2V\right\}, \quad (37)$$

$$a^{-2}h_{AB}\varphi_{,\eta}^A\varphi_{,\eta}^B = 0, \quad (38)$$

$$(2\mathcal{H}' + \mathcal{H}^2 + \epsilon) = \kappa\left\{-\frac{1}{2}h_{AB}\varphi_{,\eta}^A\varphi_{,\eta}^B + a^2V\right\}. \quad (39)$$

The chiral fields equations (6) become

$$2\mathcal{H}\varphi_A' + \left(h_{AB}\varphi'^B\right)' - \frac{1}{2}h_{BC,A}\varphi'^B\varphi'^C + a^2V_{,A} = 0. \quad (40)$$

4.2 Perturbed Einstein equations

To deals with structure formation we count the scalar type perturbations and choose the longitudinal gauge with the line element

$$ds^2 = a^2(\eta)\left\{(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\gamma_{ij}dx^i dx^j\right\}. \quad (41)$$

Using the perturbations of the energy-momentum tensor δT_ν^μ (37) and the perturbations of the gravitational field in the longitudinal gauge (41) (see the equations (4.15) in review [27]) one can derive the perturbed

Einstein equations

$$\Delta\Phi - 3\mathcal{H}\Phi' - \Phi(2\mathcal{H}^2 + \mathcal{H}') + 4\epsilon\Phi = \frac{\kappa}{2} \left\{ \langle \delta\varphi_{,\eta}\varphi_{,\eta} \rangle + \delta\varphi^C \left[\frac{1}{2}h_{AB,C}\varphi_{,\eta}^A\varphi_{,\eta}^B + a^2V_{,C} \right] \right\}, \quad (42)$$

$$(\mathcal{H} + \Phi')_{,i} = \frac{\kappa}{2}h_{AB}\delta\varphi_{,i}^A\varphi_{,\eta}^B, \quad (43)$$

$$\Phi'' + 3\mathcal{H}\Phi' + \Phi(2\mathcal{H}^2 + \mathcal{H}') = \frac{\kappa}{2} \left\{ \langle \delta\varphi_{,\eta}\varphi_{,\eta} \rangle + \delta\varphi^C \left[\frac{1}{2}h_{AB,C}\varphi_{,\eta}^A\varphi_{,\eta}^B - a^2V_{,C} \right] \right\}, \quad (44)$$

where $\langle \delta\varphi_{,\eta}\varphi_{,\eta} \rangle = h_{AB}\delta\varphi_{,\eta}^A\varphi_{,\eta}^B$.

Actually, one can use only two perturbed Einstein equations, because the linear combination of the two equations (43), (44) and the consequence of the background equations (37)-(39) as well as the fields equations (40) lead to the third equation (42).

4.3 Perturbed chiral fields equations

To understand the influence of the internal (target) space on the cosmological perturbations let us insert the metric component (41) into the chiral fields equations (6). In general case the result is too long to be presented in this review, therefore let us restrict ourself by the case of a spatially flat Universe assuming $\epsilon = 0$ in (86). Finally, the chiral fields equations can be reduced to the following view

$$\begin{aligned} (\delta\varphi_A)'' - \gamma^{ij}(\delta\varphi_A)_{,ij} + 2(\delta\varphi^B)' \left\{ h_{AB}\mathcal{H} + \Gamma_{BC,A}(\varphi^C)' \right\} + \delta\varphi^C \left\{ \left[2\mathcal{H}(\varphi^B)' + (\varphi^B)'' \right] h_{AB,C} + (h_{AB,DC} - \right. \\ \left. \frac{1}{2}h_{DB,AC})(\varphi^B)'(\varphi^D)' + a^2V_{,AC} \right\} - 2\Phi \left\{ \left[h_{AB,C} - \frac{1}{2}h_{BC,A} \right] (\varphi^B)'(\varphi^C)' + h_{AB} \left[(\varphi^B)'' + \frac{1}{2}\mathcal{H}(\varphi^B)' \right] \right\} \\ - 4(\varphi^B)'h_{AB}(\Phi' - \Phi\mathcal{H}) = 0. \end{aligned} \quad (45)$$

It needs to emphasize now that for multiple-field models the chiral metric is a constant matrix. Therefore the terms containing chiral metric derivatives will vanish in equations (45). Thus the fact that the internal chiral space gives an additional income to dynamics of cosmological perturbations is proved. The following task will be to decompose the income of auxiliary chiral fields (dark sector fields) and the target space metric from the leading field (an inflaton in the most cases). Another words, it would be desirable to safe the contribution in the cosmological perturbations from inflaton and count separately additional contribution from dark sector chiral fields and from the target space metric.

5 Decomposition of the cosmological perturbations for inflaton and dark sector ones

Let us suppose that in the range with inflaton we have during inflationary stage strong or weak fields of dark sector [10]. We could not make differences between types of DS fields before equation of state analysis will be done. Also it is worth to mention about the following understanding of the very early Universe. If we suggest that one (say, inflaton) field exists, then it is easy to imagine that this field has the sense of an effective field which presents few others [30]. Namely the relation between multiplett of scalar fields with geometrical (kinetic) interaction (that is, a chiral cosmological model, as we stressed above) can be described by an effective singlet with the relations [30]

$$h_{AB}\varphi_{,\mu}^A\varphi_{,\nu}^B = \phi_{,\mu}\phi_{,\nu}, \quad W(\varphi(x^\mu)) = V(\phi(x^\mu)), \quad \varphi = (\varphi^1, \dots, \varphi^N). \quad (46)$$

Hereafter $W(\varphi(x^\mu))$ will be presented as the potential of (self)interaction for CCM when it needs to make difference from the potential of selfinteraction $V(\phi(x^\mu))$ for the scalar singlet ϕ . Thus from GR point of view for CCM and selfgravitating scalar singlet we effectively have the same type of the source of the gravitational field, but – more complicated dynamics in the case of a chiral cosmological model. Moreover from (46) one can find the restriction on the chiral metric

$$h_{AB} = \frac{W_{,A}W_{,B}}{V'^2}, \quad V'^2 \neq 0 \quad (47)$$

Our next wish is to safe the equations for cosmological perturbations for the inflationary field as they are very successful in understanding of current observations and very good suit them. To this end let us decompose the scalar product in the target space by the following way.

$$h_{AB}\varphi_{,\mu}^A\varphi_{,\nu}^B = h_{11}\varphi_{,\mu}^1\varphi_{,\nu}^1 + h_{1b}\varphi_{,\mu}^1\varphi_{,\nu}^b + h_{b1}\varphi_{,\mu}^b\varphi_{,\nu}^1 + h_{ab}\varphi_{,\mu}^a\varphi_{,\nu}^b, \quad (48)$$

where a, b, \dots takes the values $2, 3, \dots, N$. Without the loss of generality we can choose gaussian coordinates with $h_{11} = \epsilon$ where $\epsilon = 1$ corresponds to canonical scalar field, while $\epsilon = -1$ corresponds to phantom scalar

field. (About possible phantom inflation have been stressed in the work by [18]. In according with gaussian coordinates we should also set $h_{1a} = 0$.)

Let denote the leading field φ^1 as an inflationary field ϕ , that is $\varphi^1 = \phi$.

To single out the inflationary perturbations from dark sector ones let us decompose the perturbation of the gravitation field Φ into two parts:

$$\Phi(x, t) = \Phi_\phi + \Phi_\sigma = \Phi + \Sigma, \quad (49)$$

where Φ_ϕ is responsible for the perturbations, coming from the inflationary field. Using the decompositions (48) and (49) one can separate the perturbations Φ and Σ in the perturbed Einstein equations (42)-(44) and in the perturbed fields equation (45).

Einstein equations after the separation of the perturbations take the form

$$\Phi'' + \Delta\Phi = \kappa\phi'\delta\phi', \quad (50)$$

$$\Sigma'' + \Delta\Sigma = \kappa \left(h_{ab}\delta\varphi'^a\varphi'^b + \frac{1}{2}\delta\varphi^C h_{ab,C}\varphi'^a\varphi'^b \right), \quad (51)$$

$$\mathcal{H}\Phi + \Phi' = \frac{\kappa}{2}\phi'\delta\phi, \quad (52)$$

$$\mathcal{H}\Sigma + \Sigma' = \frac{\kappa}{2}h_{ab}\delta\varphi^a\varphi^b. \quad (53)$$

Hereafter we use the simplification in notations

$$\varphi'^a := (\varphi^a)', \quad \varphi''^b := (\varphi^b)''$$

to make formulas more readable.

The perturbed chiral fields equations (45) can be also decomposed and display as

$$\delta\phi'' - \delta^{ij}\delta\phi_{,ij} + \delta\phi'2\mathcal{H} + \delta\phi a^2 W_{,\phi\phi} - 4\phi'\Phi' + 2\Phi a^2 W_{,\phi} = 0, \quad (54)$$

$$-4\phi'\Sigma' + 2\Sigma a^2 W_{,\phi} + \delta\varphi^b a^2 W_{,b\phi} - \delta\varphi'^b h_{ba,\phi}\varphi'^a - \frac{1}{2}\delta\phi h_{ba,11}\varphi'^b\varphi'^a - \frac{1}{2}\delta\varphi^d h_{ba,\phi d}\varphi'^b\varphi'^a = 0, \quad (55)$$

$$\delta\phi a^2 W_{,a1} + \delta\varphi^c a^2 W_{,ac} - 4h_{ab}\varphi'^b (\Phi' + \Sigma') + 2a^2 (\Phi + \Sigma) W_{,a} + \delta\phi h_{ab,1}\varphi'^b + \delta\varphi'^b 2\Gamma_{bc,a}\varphi'^c + \quad (56)$$

$$\delta\phi \left[(2\mathcal{H}\varphi'^b + \varphi''^b) h_{ab,1} + \left(h_{ab,d1} - \frac{1}{2}h_{db,a1} \right) \varphi'^b\varphi'^d \right] + \quad (57)$$

$$\delta\varphi^c \left[(2\mathcal{H}\varphi'^b + \varphi''^b) h_{ab,c} + \left(h_{ab,dc} - \frac{1}{2}h_{db,ac} \right) \varphi'^b\varphi'^d + h_{ab,1c}\phi'\varphi'^b \right] + h_{ab} \left(\delta\varphi''^b - \delta^{ij}\delta\varphi^b_{,ij} \right) + 2\mathcal{H}\delta\varphi'^b h_{ab} = 0. \quad (58)$$

The decomposition made with the aim to keep the equations for the gravitational perturbation Φ (106), (51) and (54) in the same form as for the scalar field (inflaton) perturbations in the longitudinal gauge (compare with the formulas (6.40-42), (6.46) in [27]).

6 Decomposition in the two-component CCM

Let us consider the application of decomposition of perturbations for the two component chiral cosmological model. Namely, let $\varphi^1 = \phi$ is the inflaton, $\varphi^2 = \chi$ is the auxiliary chiral field belonging to dark sector. Let us mention once again that physical sense of the dark sector field we can understand over equation of state analysis. We set $h_{11} = 1$, $h_{12} = 0$ in accordance with (48). Then the background equations read

$$3\{\mathcal{H}^2 + \epsilon\} = \kappa\left\{\frac{1}{2}\phi'^2 + \frac{1}{2}h_{22}\chi'^2 + a^2V\right\}, \quad (59)$$

$$a^{-2}[\phi'\phi_{,i} + h_{22}\chi'\chi_{,i}] = 0, \quad (60)$$

$$(2\mathcal{H}' + \mathcal{H}^2 + \epsilon) = \kappa\left\{-\frac{1}{2}\phi'^2 - \frac{1}{2}h_{22}\chi'^2 + a^2V\right\}. \quad (61)$$

The chiral fields equations (40) are

$$\phi'' + 2\mathcal{H}\phi' - \frac{1}{2}h_{22,\phi}\chi'^2 + a^2V_{,\phi} = 0 \quad (62)$$

$$(h_{22}\chi')' + 2\mathcal{H}h_{22}\chi' - \frac{1}{2}h_{22,\chi}\chi'^2 + a^2V_{,\chi} = 0. \quad (63)$$

Let us suggest for the sake of simplicity that h_{22} is the function depending on the leading field only: $h_{22} = h_{22}(\phi)$. Let the potential V will obey the same property: $V = V(\phi)$.

Under the assumptions above one can obtain the consequences from Einstein equations (59)-(61):

$$V = \frac{(\mathcal{H}' + 2\mathcal{H}^2)}{\kappa a^2}, \quad (\phi')^2 + h_{22}(\chi')^2 = \frac{2(\mathcal{H}^2 - \mathcal{H}')}{\kappa}. \quad (64)$$

The equations (64) give us possibility to apply the method of chiral space metric deformation [23], [3] for construction solutions for cosmological perturbations. The general scheme can be described as follow. One should specify the regime of scalar factor evolution $a = a(\eta)$, then from the first equation of (64) one can find $V = V(\eta)$ as well as the linear combination of the left hand side of the second equation of (64). Using (63) one can find the relation between inflationary field ϕ and a chiral metric component h_{22} :

$$(\phi')^2 = \mathcal{F}(\eta) - \frac{C_1}{h_{22}a^4}, \quad \mathcal{F}(\eta) = \frac{2}{\kappa} (\mathcal{H}^2 - \mathcal{H}'). \quad (65)$$

Here $C_1 = \text{const}$ is the integrating constant.

As it was pointed out earlier the inflationary field perturbations exactly extracted from the perturbed CCM equations. In the case under consideration one can derive the equation for auxiliary dark sector field perturbations. The equation is simplified if make the additional suggestion about vanishing of the second term in r.h.s. of (51). To this end we can restricted consideration for the time when chiral metric component h_{22} takes it maximum value. Then the equation for the dark sector field (51) simplified to the view:

$$\Sigma'' - \Delta\Sigma + 6\mathcal{H}\Sigma' + 2\Sigma(\mathcal{H}' + 2\mathcal{H}^2) = 0. \quad (66)$$

That is the equation above entirely determines the gravitational perturbation Σ from the dark sector field χ .

Let us transform the equation (66) to more suitable form. For this purpose we introduce new variable N by the following manner

$$\Sigma = \frac{N}{a^3(\eta)}, \quad N = N(\eta, x^i). \quad (67)$$

The equation (66) then reduced to the following one

$$N'' - \Delta N - N(5\mathcal{H}^2 + \mathcal{H}') = 0. \quad (68)$$

Let us try to find the solutions in the standard form $N = Y(\eta) \exp\{i\vec{k}\vec{x}\}$. The equation (68) takes the form

$$Y'' + Y(\eta)k^2 + Y(\eta)(5\mathcal{H}^2 + \mathcal{H}') = 0. \quad (69)$$

This equation can be easily solved in the shortwave approximation, when $k^2 \gg (5\mathcal{H}^2 + \mathcal{H}')$. The solution is

$$Y = \pm \frac{C}{k} \sin k(\eta + \eta_0) \quad (70)$$

Finally, using the relation (67), one can obtain

$$\Sigma = \pm \frac{C}{a^3 k} \sin k(\eta + \eta_0) \exp\{i\vec{k}\vec{x}\} \quad (71)$$

This result can be also presented in the form

$$\Sigma = \pm \frac{C}{a^3 k} \frac{1}{2i} \left(e^{\{ik(\eta+\eta_0)\}} - e^{\{-ik(\eta+\eta_0)\}} \right) \exp\{-i\vec{k}\vec{x}\} \quad (72)$$

Let us compare obtained solution for dark sector field perturbations with inflaton ones, described by formula

$$\Phi = \pm \frac{C_1}{ak} \frac{1}{2i} \left(e^{\{ik(\eta+\eta_0)\}} - e^{\{-ik(\eta+\eta_0)\}} \right) \exp\{-i\vec{k}\vec{x}\}. \quad (73)$$

We can conclude that dark sector field perturbations decay faster (if $a > 1$) then the inflaton ones during expansion of the Universe.

7 Decomposition of perturbations for power law Universe expansion

The background solution for the power law inflation $a = a_0 t^m$, $m > 1$ can be found by the fine turning method and has the form [34], [35].

$$\chi = 2C_0 \int \frac{dt}{h_{22}(\phi(t)) t^{3m}}, \quad (74)$$

$$\phi = \int \sqrt{\frac{2m}{\kappa t^2} - \frac{C_0^2}{h_{22}(\phi(t)) t^{6m}}} dt, \quad (75)$$

$$W(\phi(t)) = \frac{m(3m-1)}{\kappa t^2}. \quad (76)$$

General case of study small perturbations is too difficult for investigation. Therefore we choose the simplification with the time dependence for the chiral metric in the form $h_{22}(t) = \frac{2}{A}t^{2-6m}$, where A is a positive constant: $A > 0$. From this suggestion one can find

$$h_{22}(\phi) = \frac{2}{A} \exp\{(2-6m)\sqrt{\frac{\kappa}{2(m-d)}}\phi\}, \quad (77)$$

$$W(\phi) = \frac{m(3m-1)}{ae} \exp\{-2\sqrt{\frac{\kappa}{2(m-d)}}\phi\}, \quad (78)$$

where $d = \kappa AC_0^2$.

New approach for solving the complete system of Einstein and fields perturbed equations was proposed by [36]. The system of above mentioned equations was reduced to the ordinary differential equation of the forth order

$$\begin{aligned} & \Phi^{(4)} + 2\frac{2m+3}{t}\Phi^{(3)} + \left(\frac{\Pi_1(m,d)}{(m-d)t^2} + 2\frac{k^2}{a_0^2}t^{-2m}\right)\ddot{\Phi} \\ & + \left(3\frac{m(2+m)(6md+m-3d)}{(m-d)t^3} + 6\frac{k^2}{a_0^2}t^{-2m-1}\right)\dot{\Phi} + \\ & \left(\frac{\Pi_2(m,d)k^2}{(m-d)a_0^2}t^{-2m-2} + \frac{k^4}{a_0^4}t^{-4m}\right)\Phi = 0, \end{aligned} \quad (79)$$

where $\Pi_1(m,d) = 3m^3 + 15m^2d - 20md + 14m^2 + 6m - 6d$, $\Pi_2(m,d) = -2m^3 - 16m^2d + 18md + 2m - 6d$, $C_0 \neq 0$. The equation (79) obeys by the property that Φ and calculated with it $\delta\chi$ are the solutions of the complete system of perturbed equations. If we know the perturbation of the gravitational field Φ , the value of $\delta\varphi$ and $\delta\chi$ can be easily found.

In the longwave approximation, when the size of nonhomogeneities is much more then the size of a horizon, one can suggest $\frac{k}{a_0 t^{m-1}} \ll 1$. Under this suggestion we obtain the Euler equation and the list of its solutions is evident: $\Phi_k = \text{const}$. The second one has the form $\Phi_k = \text{const} \cdot t^{-m-1}$. The third and forth independent solutions are depend on the value of the parameters m and d . When $0 < d < m$ the later solutions are fast decreasing ones. For example, when $m = 3$ and $d = 1$ the complete solution takes the form

$$\Phi_k = A_k + B_k t^{-4} + C_k t^{-4} \cos(2\sqrt{14} \ln t) + D_k t^{-4} \sin(2\sqrt{14} \ln t) \quad (80)$$

If we restrict ourself only by nondecreasing mode of the metric perturbation then the chiral fields dependence on time takes the form $\delta\varphi = \text{const}$, $\delta\chi \propto t^{3m-1}$, besides the relation $\frac{\delta\varphi}{\dot{\varphi}} = \frac{\delta\chi}{\dot{\chi}}$ is valid.

The growing modes of metric perturbations may appear in the case of negative values of d , i.e. during the period when the potential W has negative values.

In the shortwave approximation the solution of the equation of the forth order is

$$\Phi_k^{(1)} = \frac{\exp(\pm \frac{ik}{a_0} \frac{t^{1-m}}{1-m})}{t} \quad (81)$$

$$\Phi_k^{(2)} = \frac{\exp(\pm \frac{ik}{a_0} \frac{t^{1-m}}{1-m})}{t^{3m}} \quad (82)$$

Corresponding to the solution above the chiral fields perturbations take the form

$$\delta\chi = \pm \frac{d}{m\kappa C_0} \frac{ik}{a_0} t^{2m-1} \exp(\pm \frac{ik}{a_0} \frac{t^{1-m}}{1-m}), \quad (83)$$

$$\delta\varphi = \pm \frac{1}{m} \sqrt{\frac{2m-2d}{\kappa}} \frac{ik}{a_0} t^{-m} \exp(\pm \frac{ik}{a_0} \frac{t^{1-m}}{1-m}) \quad (84)$$

One can see, that the metric perturbations Φ of the form (81) are coincident with the case of the power law inflation with one inflaton field. Besides the perturbations of $\delta\varphi$ are differ from inflaton ones only by a constant factor and the relation

$$\frac{\delta\varphi}{\dot{\varphi}} = \frac{\delta\chi}{\dot{\chi}}$$

is valid.

Thus the components of the model under consideration are strongly coupled even in the short wave approximation.

For the fast decreasing modes (82) the perturbations of $\delta\varphi$ can be considered in the framework of used approximation as zero. While for $\delta\chi$ one can obtain the expression

$$\delta\chi = \frac{1}{\kappa C_0} \frac{ik}{a_0} t^{-m} \exp(\pm \frac{ik}{a_0} \frac{t^{1-m}}{1-m}).$$

It should be mentioned also about strong correspondence of two approach for cosmological perturbations. Solutions (81,82) are obtained by solving the equation of the forth order, while solutions (72,73) are obtained directly from decomposed equations.

8 Dark sector fields on the inflationary background

In previous section we analyzed decomposition of the chiral fields for inflaton and dark sector fields in the framework of self-gravitating NSM with the potential of (self)interaction. It may happened that the dark sector fields were formed earlier then inflation starts. In this case we may think over new scenario in which dark sector fields are very weak and do not affect for gravitational field supported by inflaton. Analogical situation was studied by Jaffe [9] who modified and extended formalism that have been developed by Veeraraghavan and Stebbins [37]. The approach was based on the perturbations produced by a "stiff source" such as these cosmic field configuration formed by Kibble mechanism. A "stiff source" evolves in the background of FRW Universe; the back reaction of gravitational perturbations onto the source is regarded to be negligible. In our approach [10], [38] we prefer to use the term "weak source" instead of "stiff source". Note that during inflation we assume the weakness of the chiral fields φ^C in respect to inflaton ϕ ; also we regard the magnitude of dark sector fields are on the same level as for perturbed inflation field $\delta\phi$.

8.1 Basic equations of the model

To describe weak dark sector fields on the inflationary background let us construct the action integral

$$S_w = \int \sqrt{-g} d^4x \left[\frac{R-2\Lambda}{2\kappa} + \frac{1}{2} \phi_{,\mu} \phi_{,\nu} g^{\mu\nu} - V(\phi) + \frac{1}{2} h_{AB}(\varphi) \varphi_{,\mu}^A \varphi_{,\nu}^B g^{\mu\nu} - W(\varphi) \right]. \quad (85)$$

Here we denoted the leading field – the inflaton as ϕ ; the chiral fields φ^C describe dark sector fields.

For the sake of simplicity let us start with two dark sector fields $\varphi^1 = \psi$, $\varphi^2 = \chi$. This case will be represented by the two component CCM and we choose the metric in gaussian coordinates

$$d\sigma^2 = d\psi^2 + h_{22}(\psi, \chi) d\chi^2.$$

To describe the physical situation we should introduce some restrictions on the potential and kinetic energies of the fields under consideration. To this end let us represent the total potential as the sum of two parts:

$$W_{tot}(\phi, \psi, \chi) = V(\phi) + W(\psi, \chi).$$

Besides we count that $W(\psi, \chi) \ll V(\phi)$, therefore $W_{tot} \approx V(\phi)$.

Similar relations we set for inflaton (leading field) kinetic energy $K_L(\phi) = \frac{1}{2} \phi_{,\alpha} \phi^{,\alpha}$ and dark sector fields kinetic energy $K_{12}(\psi, \chi) = \frac{1}{2} \psi_{,\alpha} \psi^{,\alpha} + \frac{1}{2} h_{22}(\psi, \chi) \chi_{,\alpha} \chi^{,\alpha}$:

$$K_{tot}(\phi, \psi, \chi) = K_L(\phi) + K_{12}(\psi, \chi).$$

Supposed that $K_{12} \ll K_{tot}$, we obtain $K_{tot} \approx K_L$.

After the restrictions above the kinetic energy term K_{12} and the potential $W(\psi, \chi)$ of CCM should be deleted from Einstein equations during the inflationary period. In terms of the conformal time for FRW Universe with the metric

$$ds^2 = a^2(\eta)(-d\eta^2 + \gamma_{ij} dx^i dx^j), \quad (86)$$

(where γ_{ij} being the metric of three dimensional space-like section) the equations (37), (39) take the following form

$$3\mathcal{H}^2 = \kappa (\mathcal{K}_L + a^2 V(\phi)), \quad (87)$$

$$\mathcal{H}' - \mathcal{H}^2 = -\kappa \mathcal{K}_L, \quad (88)$$

where $\mathcal{H} = \frac{a'}{a}$ and $\mathcal{K}_L = \frac{1}{2} \phi'^2$.

Dynamic equation for inflaton reduced to [27]:

$$\phi'' + 2\mathcal{H}\phi' + a^2 V_{,\phi} = 0. \quad (89)$$

Thus, obtained equations (87)-(89) represent the inflationary epoch driving by inflaton ϕ with the potential of selfinteraction $V(\phi)$. Note that the equations (87), (88) are corresponded to the Einstein equations (10), (11) with $\epsilon = 0$, $h_{11} = 1$, $h_{12} = 0$.

Dark sector fields equations under restrictions $W(\psi, \chi) \ll V(\phi)$ and $K_{12} \ll K_{tot}$, will account only gravitational interaction by means of the Hubble parameter \mathcal{H} and read

$$\psi'' + 2\mathcal{H}\psi' - \frac{1}{2}\frac{\partial h_{22}}{\partial\psi}\chi'^2 + a^2W_{,\psi} = 0, \quad (90)$$

$$h_{22}(\chi'' + 2\mathcal{H}\chi') + \frac{\partial h_{22}}{\partial\psi}\psi'\chi' + \frac{1}{2}\frac{\partial h_{22}}{\partial\chi}\chi'^2 + a^2W_{,\chi} = 0. \quad (91)$$

The simplification for the case when $h_{22} = h_{22}(\psi)$ and $W = W(\psi)$ usually give possibility to find the examples of exact solutions for the model with $h_{22} = \sin^2\psi$, $h_{22} = \psi^2$, etc. as it was shown at the section 2.2.

8.2 The dark sector fields influence on the perturbations

According with our suggestions the Einstein equations for the perturbations can be written as

$$\delta G_\mu^\nu = \kappa(\delta T_\mu^\nu + \Theta_\mu^\nu). \quad (92)$$

Here

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - V(\phi)\right), \quad (93)$$

$$\Theta_{\mu\nu} = \psi_{,\mu}\psi_{,\nu} + h_{22}\chi_{,\mu}\chi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}\psi_{,\alpha}\psi^{,\alpha} + \frac{1}{2}h_{22}\chi_{,\alpha}\chi^{,\alpha} - W(\psi, \chi)\right). \quad (94)$$

The inflationary stage of the Universe is governed by the inflaton having the perturbation of the first order $\delta\phi$: $\tilde{\phi} = \phi + \delta\phi$, where ϕ is the solution of the background equations (89). The corresponding perturbations of the energy-momentum tensor (93) δT_μ^ν are the same order of magnitude as the energy-momentum tensor for the weak source $\Theta_{\mu\nu}$ (94) for the model under investigation.

Once again we choose the longitudinal gauge with the line element (41)

$$ds^2 = a^2(\eta)\left\{(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\gamma_{ij}dx^i dx^j\right\}.$$

Here $\Phi = \Phi(\eta, \vec{r})$ is the perturbation of the metric (of the gravitational field). Now from the equation (92), both parts of which are gauge invariant [27], using the perturbed Einstein equations (42)-(44) one can obtain modified equations for longitudinal gauge

$$\nabla^2\Phi - 3\mathcal{H}\Phi' - (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa}{2}(\phi'\delta\phi' + a^2V_{,\phi}\delta\phi + \Theta_0^0), \quad (95)$$

$$\Phi' + \mathcal{H}\Phi = \frac{\kappa}{2}(\phi'\delta\phi + \Theta_*^0), \quad (96)$$

$$\Phi'' + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa}{2}(\phi'\delta\phi' - a^2V_{,\phi}\delta\phi - \Theta_*^*), \quad (97)$$

where ϕ is the background solution of the equations (87)-(89) for the inflaton; $\delta\phi = \delta\phi(\eta, \vec{r})$ is the inflaton perturbation. The components of the weak energy-momentum tensor Θ_μ^ν are $\Theta_0^0 = \rho = a^{-2}K_{12} + W$, $\Theta_*^* = \Theta_1^1 = \Theta_2^2 = \Theta_3^3 = -p = -a^{-2}K_{12} + W$.

By adding the equation (95) with (97) and using consequence of background equations (37)-(39) $\mathcal{H}^2 - \mathcal{H}' = \frac{\kappa}{2}\phi'^2$ and the relation $\Theta_0^0 + \Theta_*^* = \rho - p = 2W$, one can obtain the differential equation in the partial derivatives of the second order in respect to the function Φ

$$\Phi'' - \nabla^2\Phi + 2\Phi'\left(\mathcal{H} - \frac{\phi''}{\phi'}\right) + 2\Phi\left(\mathcal{H}' - \mathcal{H}\frac{\phi''}{\phi'}\right) + \kappa W = 0. \quad (98)$$

Here ϕ and \mathcal{H} can be obtained from background equations for the inflationary stage (87)-(89). The potential W reflects the property of the dark sector fields and may be tested for different HEP predictions. Afterward we can define the perturbation of the gravitational field Φ by solving the equation (98) and corresponded perturbation of the inflaton $\delta\phi$ from (96):

$$\delta\phi = \frac{2}{\kappa\phi'}(\Phi' + \mathcal{H}\Phi). \quad (99)$$

Note that the inflaton perturbation $\delta\phi$ should satisfy the perturbed dynamic equation

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2V_{,\phi\phi}\delta\phi = 0. \quad (100)$$

Now we are ready to investigate the solution for inflationary Universe with exponential scale factor, which is very important for inflationary epoch analysis.

8.3 The solutions on exponential inflation background

Let us start from the given scale factor as function on time. In according with the fine tuning method [39] we can define the shape of the potential V and the dynamics of the inflationary field ϕ . It is clear that this method can be applied for the model in terms of conformal time. Therefore we set the scale factor as the exponent of the cosmic time t : $a(t) = a_s e^{h_* t}$ (a_s, h_* – constants) and make conversion to conformal time η . For the conformal time we obtain $a(\eta) = -\frac{1}{h_* \eta}$, $\eta = -\frac{e^{-h_* t}}{a_s h_*}$. The solutions of the model equations (87)-(89) are

$$\mathcal{H}(\eta) = -\frac{1}{\eta}, \quad \phi = \text{const}, \quad V(\phi) = \text{const}. \quad (101)$$

Note the restriction on the conformal time η : $\eta \in (-1/[a_s h_*], 0)$, which is corresponded to variation of the cosmic time t : $t \in (0, \infty)$.

Taking into account the solutions (101) the modified equations for perturbations (95)-(97) may be reduced to

$$\Phi'' - \nabla^2 \Phi + \kappa W(\eta) = 0. \quad (102)$$

Here we follow by the procedure suggested in [27] when we consider the consequences of perturbed gravitational equations instead of dynamical fields equations.

It was discussed in the Introduction about pure data about kinetic and potential interactions of the dark sector fields. Therefore we will study possible simple assumptions for h_{22} and W to obtain the solutions for cosmological perturbations.

Let us set the kinetic interaction h_{22} by the dependence on ψ : $h_{22} = \psi^2$ and assume the dependence of the potential W only on ψ by the following Higgs-type manner [38]

$$W(\psi) = \frac{h_*^2 \gamma^2}{4\beta^2} \psi^4 + h_*^2 \psi^2 + W_*. \quad (103)$$

For this type potential the solutions for the background dark sector fields evolution read:

$$\psi = \beta \eta, \quad \chi = \gamma \eta + \text{const}.$$

This solutions give us possibility to present the potential and kinetic energy in terms of conformal time η

$$W(\eta) = h_*^2 \beta^2 \eta^2 \left(1 + \frac{\gamma^2}{4} \eta^2\right) + W_*, \quad (104)$$

$$K(\eta) = \frac{1}{2} h_*^2 \beta^2 \eta^2 (1 + \gamma^2 \eta^2). \quad (105)$$

Here W_*, β, γ – integration constants Using the relation (104) we can solve the equation (102) for the longwave approximation. That is we in usual way represent the gravitational perturbation as the plane waves with the wave vector \mathbf{k} and amplitude Φ depending on η :

$$\Phi(\eta, \mathbf{x}) = \Phi(\eta) e^{i\mathbf{k}\mathbf{x}}.$$

Then the term $\nabla^2 \Phi$ can be neglected in the longwave approximation as the low wave number $k = |\mathbf{k}|$. After this procedure we can solve the equation (102) exactly

$$\Phi_1 = C_2 \eta - \frac{\kappa(h_* \beta \gamma)^2}{120} \eta^6 - \frac{\kappa(h_* \beta)^2}{12} \eta^4 - \frac{\kappa W_*}{2} \eta^2. \quad (106)$$

Now we can compare the result (106) with the solution for the inflaton perturbations

$$\Phi^{(0)} = C_2 \eta, \quad \delta\phi^{(0)} = C_1 \eta^3 + C_3. \quad (107)$$

without the dark sector fields with the energy momentum tensor Θ_μ^ν . Note that this solution will be agreed with the equation (96) if we require $W_* = 0$, $\beta \ll 1$ and $\beta \rightarrow 0$ $\eta \simeq \eta_{infl}$.

More solutions, analysis of their physical properties and graphical illustrations were discussed in the works [10], [38]. Note that we used the direct integration of the equation (98) instead of the reducing this equation to new variable [27] $u = \frac{a}{\phi} \Phi$. The reason is the specific property of the investigated solution where $\phi' = 0$ in the denominator. To get round this problem let us consider the power law inflation.

8.4 The solutions on power law inflation background

Let us choose the scale factor in the power law expansion form $a = a_* t^m$ for the cosmic time t . Corresponded evolution for the conformal time η may be represented as $a = a_s \eta^\alpha$ with $\alpha = \frac{m}{1-m}$. Suggesting the inflation expansion we have to set $m > 1$ implying $\alpha < -1$. Using the relations between cosmic and conformal times: $\eta = \frac{t^{1-m}}{a_s(1-m)}$, $t = [a_s(1-m)\eta]^{\frac{1}{1-m}}$ one can see that when $t \rightarrow 0, \eta \rightarrow -\infty$ and when $t \rightarrow \infty, \eta \rightarrow 0$. The background solutions of the equations (87)-(89) are

$$\mathcal{H}(\eta) = \frac{\alpha}{\eta}, \quad \phi = \sqrt{\frac{2}{\kappa}} \alpha(\alpha+1) \ln \eta + \phi_0, \quad V(\phi) = \frac{\alpha(2\alpha-1)}{a_s \kappa} \exp \left(-\frac{(\alpha+2)(\phi-\phi_0)}{\sqrt{\frac{2}{\kappa}} \alpha(\alpha+1)} \right). \quad (108)$$

Taking into account the solutions (108) one can reduce the main equation for the gravitational perturbation (98) to the following form

$$\Phi'' - \nabla^2 \Phi + 2\Phi' \left(\frac{\alpha+1}{\eta} \right) + \kappa W = 0. \quad (109)$$

The potential W of interaction between dark sector fields ψ and χ can not be suggested from observation data. Therefore we can consider various functional dependence W on ψ follow by intuition or analogies from particle physics for example. Let as once again set the kinetic interaction h_{22} by the dependence on ψ : $h_{22} = \psi^2$ and assume that the potential W is the function only on ψ by the following type

$$W = -K_1^2 \psi^{\lambda_1} + K_2^2 \psi^{\lambda_2} + W_0. \quad (110)$$

Here K_1, K_2, λ_1 and λ_2 are constants which should be matched with the model parameters. Taking as in previous case the linear dependence on conformal time for the dark sector field ψ : $\psi = \beta \eta$ one can obtain the solution for the second field χ . Thus the solutions for the dark sector fields are:

$$\psi = \beta \eta, \quad \chi = \chi_0 - \frac{1}{(2\alpha+1)} \frac{1}{\eta^{(2\alpha+1)}}. \quad (111)$$

With this solution it is not difficult to obtain the following values for the constants

$$K_1^2 = \frac{\beta^{2(\alpha+1)}}{a_s^2}, \quad \lambda_1 = -2\alpha, \quad K_2^2 = \frac{2C_1^2 \beta^{6\alpha}}{a_s^2(1-2\alpha)}, \quad \lambda_2 = 1-2\alpha. \quad (112)$$

Neglecting by the term $\nabla^2 \Phi$ in the long-wavelength approximation we can solve the equation (109) exactly. The gravitational perturbation is

$$\Phi = \frac{\kappa \beta^2}{6a_s^2(1-\alpha)} \eta^{2(1-\alpha)} - \frac{2\kappa C_1^2 \beta^{4\alpha+1}}{4a_s^2(1-2\alpha)(3-2\alpha)} \eta^{(3-2\alpha)} - C_2 \frac{\eta^{-(2\alpha+1)}}{(2\alpha+1)}. \quad (113)$$

It is clear that if we put suitable values for power law inflation m for cosmic time and then by calculating the power α for conformal time we obtain the potential of dark sector field W in terms of double power law combination (110). For example taking $m = 3$ one can obtain the potential of dark sector fields

$$W = -\frac{1}{a_s^2 \beta} \psi^3 + \frac{C_1^2}{2a_s^2 \beta^9} \psi^4 + W_0. \quad (114)$$

The gravitational perturbation (109) reads

$$\Phi = \frac{\kappa \beta^2}{15a_s^2} \eta^5 - \frac{\kappa C_1^2}{48a_s^2 \beta^5} \eta^6 + \frac{C_2}{2\eta^2}. \quad (115)$$

Thus we can see the influence of the expansion parameter α and dark sector field parameter β for the gravitational perturbation and the potential W . Of course if we may obtain some information about dark potential W we immediately can insert it in the equations (90)-(91) and try to solve them.

The comparison with standard solution for long-wavelength approximation (the formula (5.65) in [27])

$$\Phi^{(0)} = A \eta^{1+\alpha}, \quad A = \text{const} \quad (116)$$

stress the more crucial difference then in the case with exponential inflation. Namely there are no terms in (109) corresponding to standard solution (116) for any admitted α . Thus taking into account the dark sector fields influence for gravitational perturbation we may obtain a very different picture for structure formation.

9 Conclusions

In the present review we show the advantage of the chiral cosmological model for description of the early and later inflation. Namely CCM successively used for the dark sector fields description for the explanation of accelerated Universe expansion at the present time.

To connect the early inflation with predictions for observation we presented general formulas for cosmological perturbations from CCM and described the method of decomposition of cosmological perturbations for inflaton and dark sector ones. Based on this approach we proposed new generalized quintom model and founded out new exact solutions for it. We discussed new point of view on decomposition of CCM perturbations in the light of dark energy fields using the example of power law Universe expansion.

Let us mention that the chiral cosmological model, based on the NSM with the (self)interacting potential, contains self-interacted single-field theories and multiple scalar fields models which may be inspired by superstrings cosmology. That is the chiral cosmological model can be reduced to the models mentioned above under the special restrictions on the target space metric.

We show the new possibilities of investigation of cosmological perturbations taking into account influence of dark sector fields on inflation background.

References

- [1] E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley, Redwood City, California (1990)
- [2] E.J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006)
- [3] S.V. Chervon, *Russ.Phys.J.*, New York, v.38, 539 (1995)
- [4] S.V. Chervon *Non-linear sigma model for inflation scenarios. II*, IUCAA Preprint, IUCAA-26/93, October 1993.-14 p. (1993)
- [5] A.M. Perelomov, *Phys. Rep.* **146**, No.3, 136 (1987)
- [6] V. De'Alfaro, S. Fubini and G. Furlan *Nuovo Cim. A* **50**, 523 (1979)
- [7] G.G. Ivanov *Theor. Math. Phys.* **57** 45 (1983)
- [8] S.V. Chervon *Non-linear sigma models for inflation scenarios*, IUCAA Preprint, IUCAA- 15/92, October 1992.- 16 p. (1992)
- [9] A.H. Jaffe *Phys. Rev.* **D49**, 3893 (1994)
- [10] S.V. Chervon and O.G. Panina *Journal "Vestnik RUDN"* 4, 121 (2010)
- [11] A.L. Berkin and R.W. Hellings, *Phys. Rev. D* **49**, 12, 6442 (1994)
- [12] P.C. Aichelburg and C. Lechner, *Phys. Rev. D* **57**, 6176 (1998)
- [13] V. Sahni and L. Wang, *Phys. Rev. D* **62**, 103517 (2000)
- [14] V. Sahni, *Scalar fields models for an accelerationg universe*, ArXiv: 0912.5304 (2009)
- [15] T. Matos and L.A. Urena-Lopez *Cl. Quantum Grav.* **17** L75 (2000)
- [16] T. Matos and L.A. Urena-Lopez, *Phys. Rev. D* **63**, 063506 (2001)
- [17] T. Koivisto *Phys. Rev. D* **72**, 043516 (2005)
- [18] S. Capozziello, S. Nojiri and S.D. Odintsov *Phys. Lett. B* **632**, 597 (2006)
- [19] G-B. Zhao, J-Q. Xia, M. Li, B. Feng and X. Zhang, *Phys. Rev. D* **72**, 123515 (2005)
- [20] C.G. Bohmer, G. Cadera-Cabral, R. Zazkoz and R. Maartens, *Phys. Rev. D* **78**, 023505 (2008)
- [21] J. Valiviita, E. Majerotto and R. Maartens, *JCAP* 0807:020 (2008)
- [22] L.P. Chimento, M. Forte, R. Lazkoz and M.G. Richarte *Phys. Rev. D* **78**, 043502 (2009)
- [23] S.V. Chervon, *Gravitation & Cosmology*, Vol.1, No.2, 91 (1995)
- [24] M.R. Setare and E.N. Saridakis *JCAP* 0809:026 (2008)
- [25] C. Van de Bruck and J.M. Weller, *Phys. Rev. D* **80**, 123014 (2009)
- [26] E.N. Saridakis and J.M. Weller *Phys. Rev. D* **81**, 123523 (2009)
- [27] V.F. Mukhanov, H.A. Feldman and R.H. Brandenberger, *Phys. Rep.* **215** , No.5, 130 (1992)
- [28] S.V. Chervon, *Exact solutions in standard and chiral inflationary models*, Proceedings of 9th Marcell Grossman Conference, Roma, 2000. World Scientific, p.1909, Pt.C.(2001)
- [29] S.V. Chervon, *Gravitation & Cosmology.* **3**, 32 (2002)
- [30] S.V. Chervon, *Gravitation. & Cosmology* **3**, 145 (1997)
- [31] S.V. Chervon and R.R. Abbyazov, (submitted for publication) (2012)

- [32] D. Baumann and H.V. Peiris *Adv. Sci. Lett.* **2**, 105 (2009)
- [33] E.M. Lifshitz, *Zh. Eksp. Teor. Fis.* **16** 587 (1946)
- [34] S.V. Chervon and L.V. Zakharova, *Gravitation & Cosmology.* **3**, No.4, 317 (1997)
- [35] S.V. Chervon and N.A. Koshelev, *Gravitation. & Cosmology* **9**, No.3, 196 (2003)
- [36] N.A. Koshelev, *Russ.Phys.J.*, New York, No.12 (2003)
- [37] S. Veeraraghavan and A. Stebbins *Astrophys. J.* **365** 37 (1989)
- [38] O.G. Panina and S.V. Chervon *On the pre-inflationary dark sector fields influence on the cosmological perturbations*, Proc. of Sci. The XXth International Workshop HEP and QFT, Sept.-24 – Oct.1, 2011; <http://pos.sissa.it> , (2011)
- [39] S.V. Chervon, V.M. Zhuravlev and V.K. Shchigolev *Phys. Lett. B* **398**, 269 (1997)